



Lösungen

Kapitel 1: Grundrechenarten

1) (a) $a(a+1)$ (b) $ab(a+b)$ (c) $a(b+c-d)$ (d) $(a-1)(b-c)$

(e) $4b(2a+5b)$ (f) $(a+4)(2b+3)$ (g) $(2+3b)(a+1)$

2) (a) $3b+2c$ (b) $4a+2b+3c$

3) (a) $18a^2 - ab - 4b^2$ (b) $2(bc - ad)$ (c) $4a^2b^2$

4) (a) $(x+y)^2$ (b) $(7x+y)^2$ (c) $(4x-2y)^2$ (d) $(2u+3v)(2u-3v)$

(e) $(2u+5v)^2 - u^2 = (2u+5v+u)(2u+5v-u) = (3u+5v)(u+5v)$

5) (a) $\square = 5$ (b) $\square = 2uv$, $\bigcirc = 9w^2$ (c) $\square = y$, $\bigcirc = 0.25x^2$, $\nabla = y^2$

(d) $\square = 7vw^2$, $\bigcirc = 25u^4$

6) (a) $b = a^2$ und $a < 1 \rightarrow b < a$ (b) Kürzen mit 11 $\rightarrow a = b$

(c) $b = -\frac{11}{10} \rightarrow |a| < |b| \rightarrow b < a$

7) (a) $\frac{8}{3}$ (b) $\frac{1}{3}$ (c) $\frac{9a}{2b}$, $a, b \neq 0$ (d) $\frac{y(3x-z)}{x(4z+2y)}$, $x \neq 0$, $y \neq -2z$

(e) $\frac{7ab-1}{2+3ab}$, $a, b \neq 0$, $a \cdot b \neq -2/3$

8) (a) 2 (b) $\frac{9}{14}$ (c) $\frac{25}{24}$ (d) $\frac{-5}{3(a+1)}$, $a \neq -1$

(e) $\frac{3a-14b^2+3a^2b^2}{6ab}$, $a, b \neq 0$ (f) $\frac{y}{z+2}$, $x \neq 0$, $z \neq -2$

9) (a) $\frac{1}{8}$ (b) $\frac{100}{63}$ (c) 2 (d) $\frac{1}{4}$ (e) 2

- 10) Der Scheich hat nicht 100% seines tierischen Vermögens aufgeteilt, da $\frac{1}{2} + \frac{1}{3} + \frac{1}{9} = \frac{17}{18} < 1$. Es wurden also $17/18$ von $n + 1$ Kamelen aufgeteilt. Das restliche $1/18$ ist das übrig bleibende Kamel. Also gilt $n + 1 = 18$ und die vererbte Anzahl der Kamele war 17.

Kapitel 2: Potenzen, Wurzeln, Logarithmen

- 11) (a) a^{-4} (b) $-a^6$ (c) $(b - a)^3$ (d) $-b^4$
- 12) (a) -81 (b) -125 (c) $-\frac{1}{8}$ (d) $\frac{4}{9}$
- 13) (a) a^{n+3} (b) $9a^n c^7$ (c) $ab^2 c^{-1} x^{-1} z^{-2}$
- 14) (a) 1 (b) 1 (c) -1 (d) 0 (e) 2 (f) 5 (g) 7
- 15) (a) 3 (b) 0.2^5 (c) $a^{3/2 \cdot 1/3 \cdot 1/2} = a^{1/4} = \sqrt[4]{a}$
- (d) $\sqrt[3]{\frac{a^6}{3} + \sqrt{\frac{4}{9}a^{12}}} = \sqrt[3]{\frac{a^6}{3} + \frac{2a^6}{3}} = \sqrt[3]{a^6} = a^2$
- 16) (a) $-5\sqrt{3}$ (b) $4\sqrt{2}$ (c) 105
- 17) (a) 1 (b) 2 (c) 4 (d) -2 (e) $1/4$ (f) e (g) 1000 (h) $1/4$ (i) 4 (j) 3
- 18) (a) $-1/2$ (b) $5/7 \ln a$ (c) $-0.5 \lg(a) - \lg(b)$
- 19) (a) $\ln(2)$ (b) $4 \lg(a) = \lg(a^4)$ (c) $\ln(x)$
- 20) Bestand nach 10 Jahren = $0.96^{10} = 0.6648 \approx 2/3$
- 21) $6a^2 = 96 \rightarrow a^2 = 16 \rightarrow a = 4$
- 22) $20000 \cdot 0.75^3 = 8437.50$

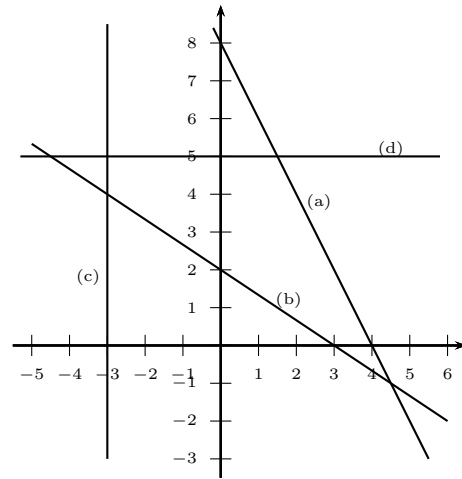
Kapitel 3: Gleichungen

- 23) (a) $x = 3$ (b) $x = -2$ (c) $x = 5$
- 24) (a) $x = 3$ oder $x = -3$ (b) $x = 2$ oder $x = -3$
(c) $x = 0$ oder $x = 5$ (d) nur $x = 1$
- 25) (a) keine Lösung (b) $x = 12$ oder $x = -4$ (c) $x = 0.5$ oder $x = 1.5$
- 26) (a) $x \neq -2.5$, Lösung $x = 2$ (b) $x \neq -4, 3$, Lösung $x = -7.5$
(c) $x \neq 6, -2$, Lösung $x = 0.6$
- 27) (a) $x \geq 1/4$, Lösung $x = 1$ (b) $x \geq 7$, Lösungen $x = 7$ oder $x = 10$
(c) $x \geq 5/9$, Lösung $x = 1$ und nicht $x = 6$ (Probe!)
- 28) (a) $x > 0$, Lösung $x = 50$ (b) $x > -5/11$, Lösung $x = 5$
(c) $|x| > \sqrt{8}$, Lösung $x = \pm 3$ (d) $x > 1$, Lösung $x = 2$
- 29) (a) $x = 2 \lg(3)$ (b) $x = -8$ (c) $x = 3$
- 30) (a) $u = x^2 \rightarrow u^2 - 4u - 45 = 0 \rightarrow u = -5, 9 \rightarrow x = \pm 3$ ($x^2 = u = -5$ geht nicht)
(b) $u = e^x \rightarrow u = 1, 2 \rightarrow x = 0, \ln(2)$
(c) $u = \lg(x) \rightarrow u = 1, 2 \rightarrow x = 10, 100$
- 31) (a) $x = 8, y = 3$ (b) $x = -2, y = 4$
- 32) (a) $x = 0, y = 10$ oder $x = -2, y = 30$ (b) $x = 1, y = 0$ oder $x = 5, y = 8$

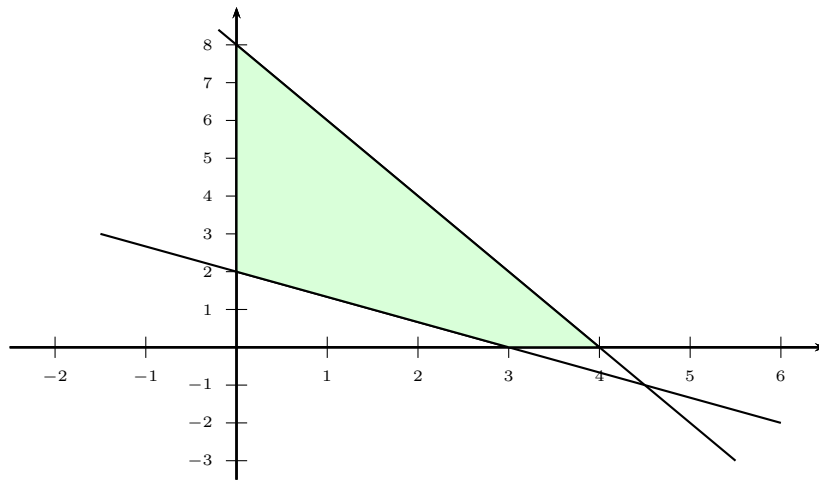
Kapitel 4: Funktionen

33)

- (a) $x = 4$ und $y = 8$
- (b) $x = 3$ und $y = 2$
- (c) $x = -3$ und y nicht
- (d) x nicht und $y = 5$

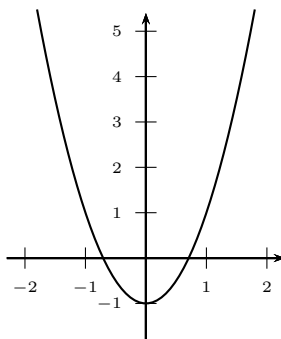


34)



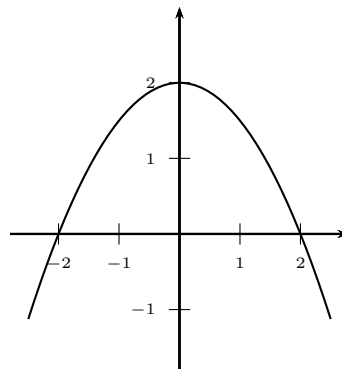
35) (a) $y = 2(x - 0)^2 - 1$
 $S = (0, -1)$

Nullstellen $x = \pm \frac{1}{\sqrt{2}}$



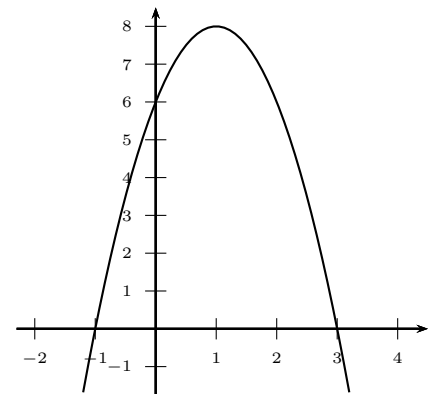
(b) $y = \frac{-1}{2}(x - 0)^2 + 2$
 $S = (0, 2)$

Nullstellen $x = \pm 2$



(c) $y = -2(x - 1)^2 + 8$
 $S = (1, 8)$

Nullstellen $x = -1, x = 3$



36) (a) $g(x) = 2x + 1, \quad f(x) = x^3, \quad D = \mathbb{R}$

(b) $g(x) = 4 - x^2, \quad f(x) = \ln(x), \quad D = (-2, 2)$

(c) $g(x) = \frac{x-1}{x-2}, \quad f(x) = \sqrt{x}, \quad D = (-\infty, 1] \cup (2, \infty)$

37) (a) $x^2 + x - 6 = 0 \rightarrow x_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + 6} = \frac{-1 \pm 5}{2} \rightarrow N = \{2, -3\}$

(b) $\ln(10 - x^2) = 0 \rightarrow 10 - x^2 = 1 \rightarrow x^2 = 9 \rightarrow N = \{-3, 3\}$

(c) $x^2 - 1 = 0 \rightarrow N = \{-1, 1\}$

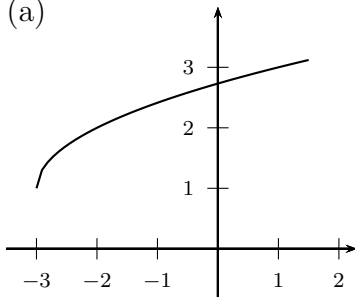
(d) $u := x^2 \rightarrow u^2 - 17u + 16 = 0 \rightarrow u \in \{1, 16\} \rightarrow N = \{-1, 1, -4, 4\}$

38) (a) $y = 5x + 3 \rightarrow 5x = y - 3 \rightarrow x = \frac{y-3}{5} \quad f^{-1}: \mathbb{R} \rightarrow \mathbb{R}, \quad f^{-1}(x) = \frac{x-3}{5}$

(b) $y = \frac{2x+1}{x-1} \rightarrow xy - y = 2x + 1 \rightarrow xy - 2x = y + 1 \rightarrow x(y-2) = y + 1 \rightarrow x = \frac{y+1}{y-2}$

$$f^{-1}: \mathbb{R} \setminus \{2\} \rightarrow \mathbb{R} \setminus \{1\}, \quad f^{-1}(x) = \frac{x+1}{x-2}$$

39) (a)



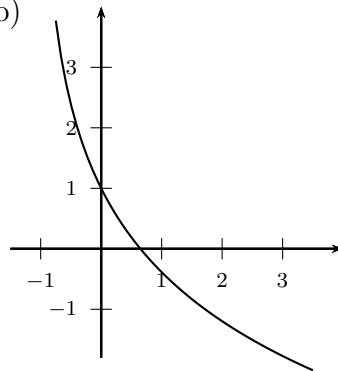
$$D = \{x : x \geq -3\}$$

$$W = \{y : y \geq 1\}$$

x -Achse: -

$$y\text{-Achse: } y = \sqrt{3} + 1$$

(b)



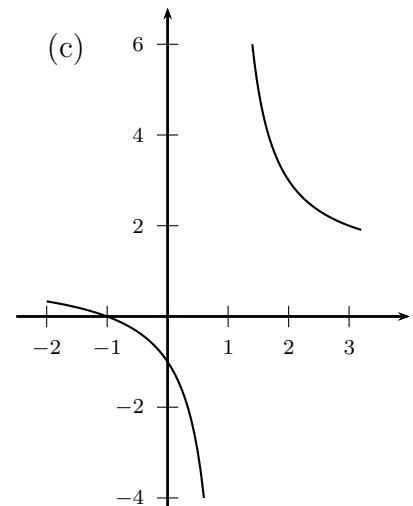
$$D = \{x : x > -1\}$$

$$W = \mathbb{R}$$

x -Achse: $x = \sqrt{e} - 1$

$$y\text{-Achse: } y = 1$$

(c)

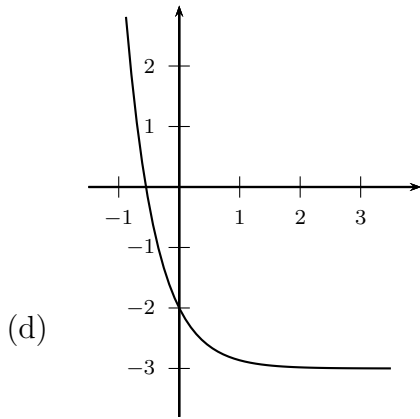


$$D = \{x : x \neq 1\}$$

$$W = \{y : y \neq 1\}$$

x -Achse: $x = -1$

$$y\text{-Achse: } y = -1$$

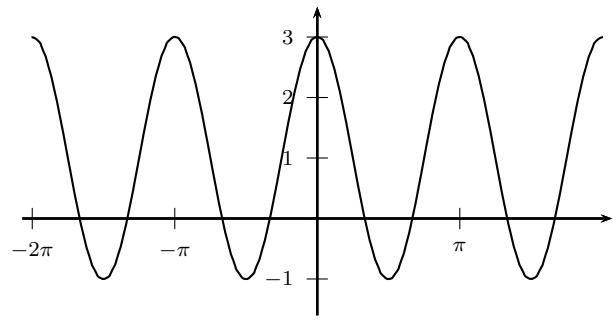


$$D = \mathbb{R}$$

$$W = \{y : y > -3\}$$

$$x\text{-Achse: } x = -\ln(3)/2$$

$$y\text{-Achse: } y = -2$$



$$D = \mathbb{R}$$

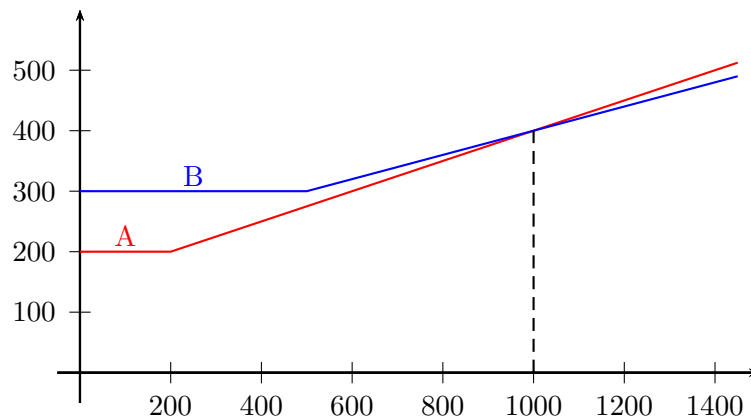
$$W = [-1, 3]$$

$$x\text{-Achse: } x = \pm\pi/3 + k \cdot \pi$$

$$y\text{-Achse: } y = 3$$

40) (a)
$$A(x) = \begin{cases} 200, & 0 \leq x \leq 200 \\ 200 + 0.25 \cdot (x - 200) = 0.25x + 150, & x > 200 \end{cases}$$

$$B(x) = \begin{cases} 300, & 0 \leq x \leq 500 \\ 300 + 0.2 \cdot (x - 500) = 0.2x + 200, & x > 500 \end{cases}$$



(b) $0.25x + 150 = 0.2x + 200 \rightarrow 0.05x = 50 \rightarrow x = 1000$

Also: $x < 1000 \rightarrow$ Tarif A und $x > 1000 \rightarrow$ Tarif B

41) (a) $x = x(p) = 6000 - 5p$

(b) $E(x) = x \cdot p(x) = 1200x - 0.2x^2$

(c) $E(p) = x(p) \cdot p = 6000p - 5p^2$

(d) $G(x) = E(x) - K(x) = 1200x - 0.4x^2 - 500000$

(e) $500 \leq x \leq 2500$

Kapitel 5: Ableitungen

42) (a) $6x^5 + 10x^4 - 12x^2 + 1$ (b) $\ln(x) + 1$ (c) $\cos^2(x) - \sin^2(x)$

(d) $\frac{x^2 + 2x - 1^2}{(x + 1)}$ (e) $\frac{1 - \ln x}{x^2}$

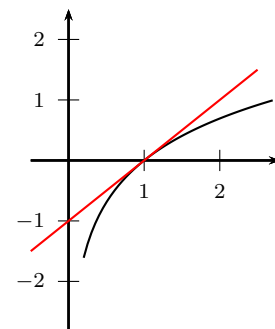
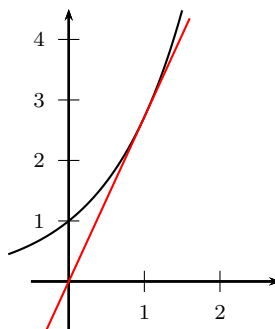
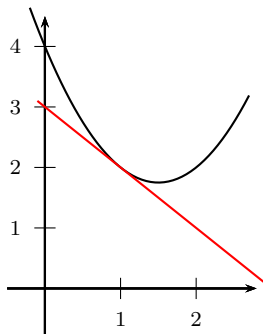
43) (a) $u(x) = 3x + 5$, $v(u) = \sin(x)$, $f'(x) = 3 \cos(3x + 5)$

(b) $u(x) = x^2 + x + 1$, $v(u) = u^9$, $f'(x) = (2x + 1) \cdot 9 \cdot (x^2 + x + 1)^8$

44) (a) $t(x) = 3 - x$

(b) $t(x) = e \cdot x$

(c) $t(x) = x - 1$



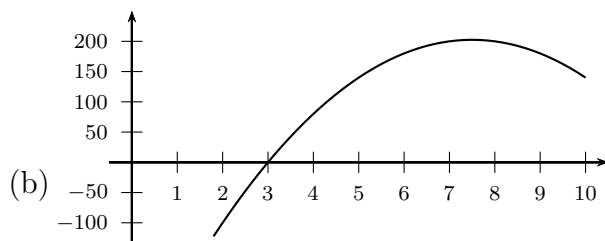
45) (a) $x = \pm 1$ (b) $x = 0$ und $x = 6$ (c) $x = \pm 2$

46) (a) $f'(x) = \frac{1}{9} - \frac{400}{x^2} = 0$, für $x = \pm 60$, also 60 km/h

(b) $K(x) = 50 + 18 \cdot \frac{700}{x} + 1.5 \cdot 7 \cdot \left(\frac{x}{9} - 3 + \frac{400}{x} \right) = 18.5 + \frac{16800}{x} + \frac{7x}{6}$

(c) $K'(x) = -\frac{16800}{x^2} + \frac{7}{6} = 0$, für $x = \pm 120$, also 120 km/h

47) (a) $G(x) = \text{Erlös} - \text{Kosten} = \text{Menge} \cdot \text{V-Preis} - \text{Kosten} = \text{Menge} \cdot (\text{V-Preis} - \text{E-Preis})$
 $= (120 - 10x) \cdot (x - 3) = -10x^2 + 150x - 360 = -10((x - 7.5)^2 - 20.25)$



$G'(x) = -20x + 150$

(c) $= 0$, für $x = 7.5$

Also: 7,50 €

48) (1) 1e, 2f, 3d, 4a, 5b, 6c

(2) 1a, 2e, 3c, 4b, 5f, 6d

(3) 1d, 2f, 3e, 4c, 5b, 6a