Internationale Elektrotechnik - Olympiade

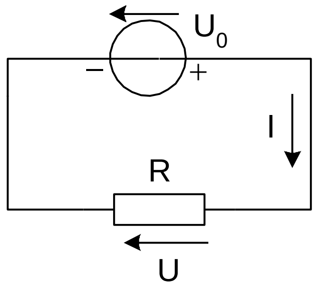
NEISSE – ELEKTRO

Formulas

english edition

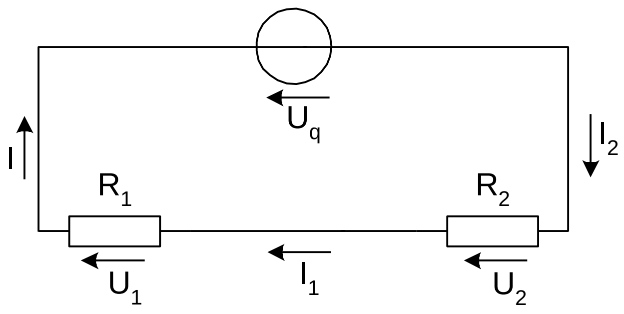
<u>**version 2017-03**</u>

Simple Circuit

electrical voltage U	$U = \varphi_1 - \varphi_2$	φ_1 electrical potential of point 1 φ_2 electrical potential of point 2
electrical current I	$I = \frac{dQ}{dt}$ under the condition of a stationary current (I = constant) apply to: $I = \frac{Q}{t}$	Q electric charge t time
current density S	$S = \frac{I}{A}$	
electrical resistor R	$R = \frac{U}{I}$	
electrical conductance G	$G = \frac{1}{R}$	
electrical power P	$P = U \cdot I$	
electrical work W	$W = P \cdot t$	U_0 voltage of the voltage source
OHM`s law	under the condition ϑ = constant apply to: $U \sim I,\ \frac{U}{I}$ = constant	
assessment of resistor	under the condition ϑ = constant apply to: $R = \frac{\rho \cdot l}{A}$	
electrical conductivity $\gamma\,(\kappa)$	$\gamma = \frac{1}{\rho}$	ϑ temperature ρ specific electrical resistance
influence of the temperature on the electrical resistor	$\Delta R = \alpha \cdot R_{20} \cdot \Delta\vartheta$ with $\Delta\vartheta = \vartheta - 20°C$ $R_\vartheta = R_{20}(1 + \alpha \cdot \Delta\vartheta)$	l length of the conductor A cross-section area R_ϑ resistor by the temperature ϑ R_{20} resistor by 20 °C α temperature coefficient

Direct current circuits

Series connection of resistors



$$I = I_1 = I_2 = \ldots = I_n$$

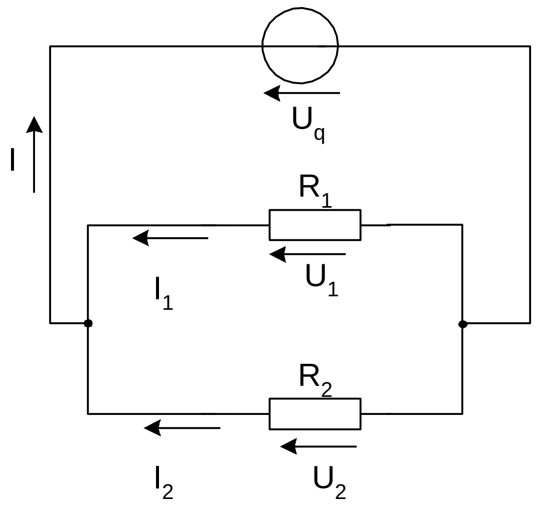
$$U_q = U_1 + U_2 + \ldots U_n$$

$$R = R_1 + R_2 + \ldots R_n$$

potential divider rule:

$$\frac{U_1}{U_2} = \frac{R_1}{R_2} \qquad \frac{U_1}{U_q} = \frac{R_1}{R}$$

Parallel connection of resistors



$$I = I_1 + I_2 + \ldots + I_n$$

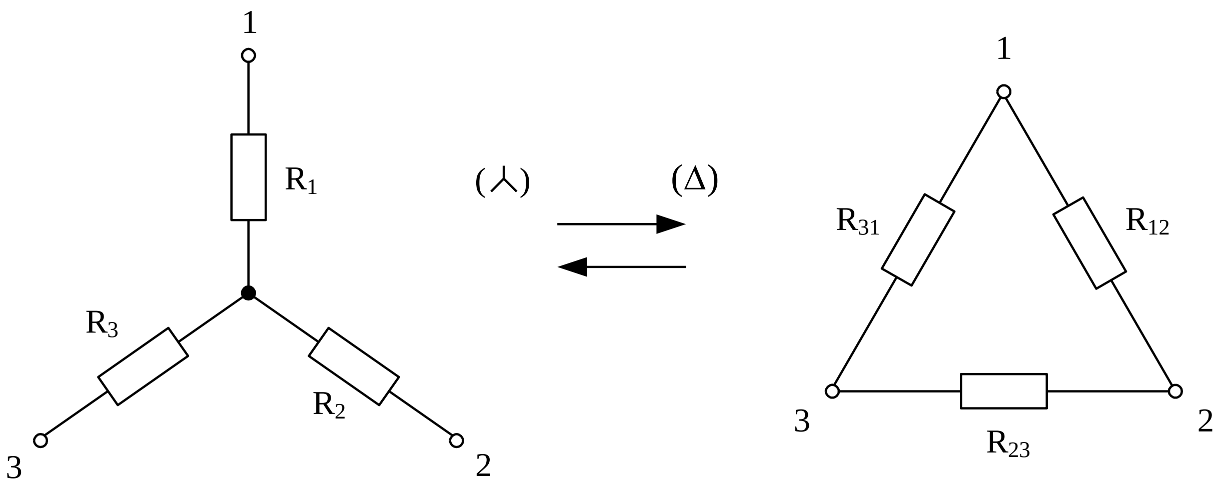
$$U_q = U_1 = U_2 = \ldots = U_n$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

current divider rule:

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} \qquad \frac{I_1}{I} = \frac{R}{R_1}$$

Network transformation



Transformation ⅄ $\longrightarrow \Delta$

$$R_{12} = R_1 + R_2 + \frac{R_1\,R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_2\,R_3}{R_1}$$

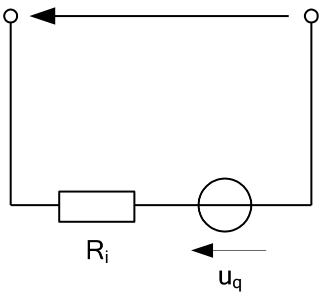
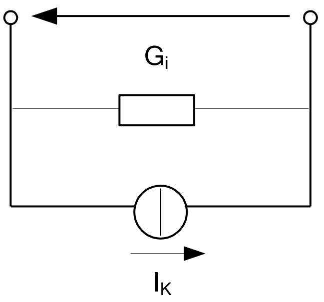
$$R_{31} = R_3 + R_1 + \frac{R_3\,R_1}{R_2}$$

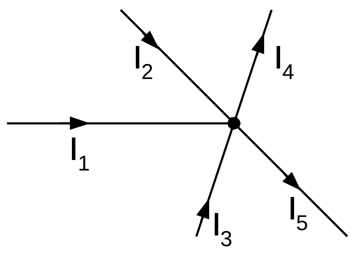
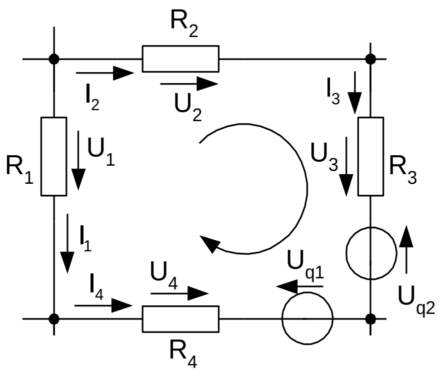
Transformation $\Delta \longrightarrow$ ⅄

$$R_1 = \frac{R_{12}\,R_{31}}{R_{12} + R_{23} + R_{31}}$$

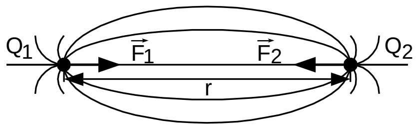
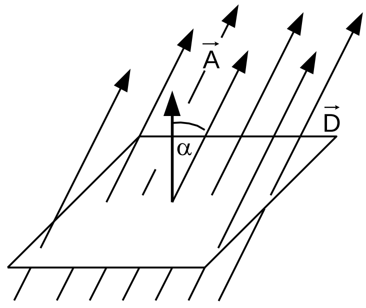
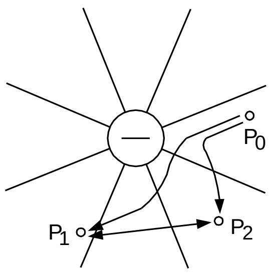
$$R_2 = \frac{R_{23}\,R_{12}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{31}\,R_{23}}{R_{12} + R_{23} + R_{31}}$$

Series connection of voltage sources	**Parallel connection of voltage sources**
$U = \sum U_q$ $R_i = \sum R_i$	$I_K = \sum I_K$ $G_i = \sum G_i$
 $R_i = \frac{U_q}{I_K}$	$R_i = \frac{1}{G_i}$ 

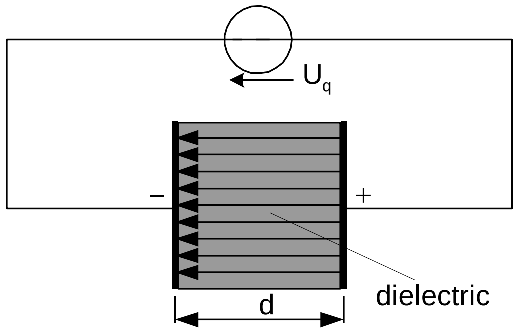
1. KIRCHHOFF's law (Junction law)	**2. KIRCHHOFF's law** (Meshwork law)
 $-I_1 - I_2 - I_3 + I_4 + I_5 = 0$	 $-U_1 + U_2 + U_3 - U_{q2} + U_{q1} - U_4 = 0$
$\sum_{k=1}^{n} I_k = 0$	$\sum_{i=1}^{n} U_i + \sum_{k=1}^{m} U_{qk} = 0$

Electrical field

electrical charge Q	$Q = N \cdot e$ $Q = \int_{t_1}^{t} I(t)\,dt$	N number of Electrones e elementary charge (p. 14) I current intensity t time
COULOMB’s law	for point charces, apply to: $F = \frac{1}{4\pi \cdot \varepsilon_0 \cdot \varepsilon_r} \quad \frac{Q_1 \cdot Q_2}{r^2}$	F force ε_0 permitivity (vacuum) (p. 14) ε_r dielectric constant r distance between the point charges 
electrical field strength E	$\vec{E} = \frac{\vec{F}}{Q}$ for homogeneous electric field, apply to: $E = \frac{U}{s}$	s distance between the points, which have the voltage drop U
electrical flux density D	$\vec{D} = \varepsilon_0 \cdot \varepsilon_r \cdot \vec{E}$ for vacuum apply to: $\varepsilon_r = 1$	
dielectric constant ε	$\varepsilon = \varepsilon_0 \cdot \varepsilon_r$	
electrical flux Ψ	$\Psi = \int \vec{D}\, d\vec{A}$ under the condition $\vec{D}$ = constant und $\vec{D} \parallel \vec{A}$ apply to: $\Psi = D \cdot A$	A area
electrical potential φ	$\varphi = \frac{W}{Q} \qquad \varphi = \int_{P_0}^{P_1} \vec{E}(s)\,d\vec{s}$	
electrical voltage U	$U = \varphi_1 - \varphi_2 \quad U = \int_{P_2}^{P_1} \vec{E}(s)\,d\vec{s}$ for homogeneous field, apply to: $U = \vec{E} \cdot \vec{s}$	W dislocation work on a charge Q in the electrical field φ_1 electrical potential in point P_1 φ_2 electrical potential in point P_2 s distance

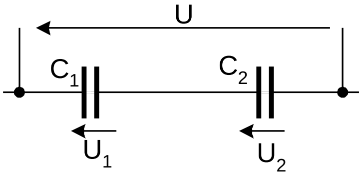
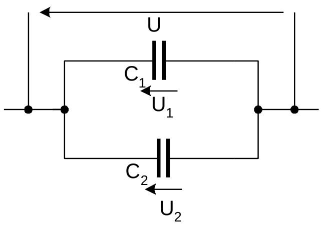
Capacitors

capacitance C of an capacitor	$C = \frac{Q}{U}$
breakdown strength E_d	$E_d = \frac{U}{d}$
electrical strength of field E of an plate capacitor	$E = \frac{U}{d}$
capacitance C of an plate capacitor and	$C = \varepsilon_0 \cdot \varepsilon_r \cdot \frac{A}{d}$
zylindrical capacitor	$C = \frac{2\pi \cdot \varepsilon_0 \cdot \varepsilon_r \cdot \ell}{\ell n \frac{r_a}{r_i}}$
energy E of the electric field (plate capacitor)	$E = \frac{1}{2} C \cdot U^2$
charge of an capacitor	$U_C = U \cdot \left(1 - e^{\left(-\frac{t}{R \cdot C}\right)}\right)$ $I = I_0 \cdot e^{\left(-\frac{t}{R \cdot C}\right)}$
discharge of an capacitor	$U_C = U \cdot e^{\left(-\frac{t}{R \cdot C}\right)}$ $I = I_0 \cdot e^{\left(-\frac{t}{R \cdot C}\right)}$
time constant τ	$\tau = R \cdot C$

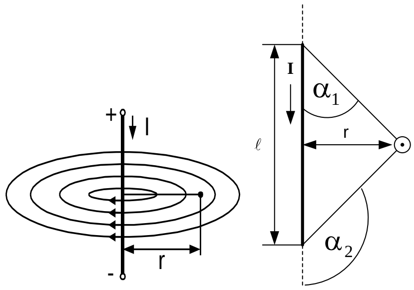
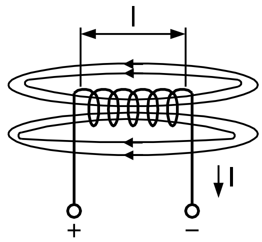
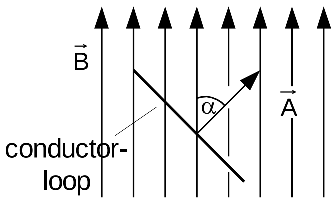
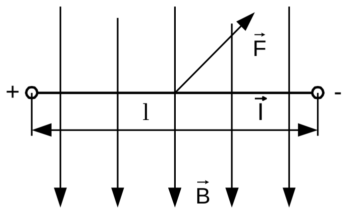


Q	charge
U	voltage
d	distance between the plates
ε_0	permittivity (vacuum) (p. 14)
ε_r	dielectric constant
A	area

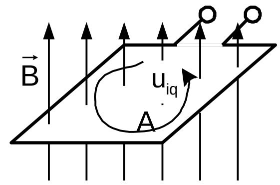
U_C	capacitor-voltage
U	charging voltage
R	OHM‘s resistor
C	capacitor
t	time
I	current intensity
I_0	current intensity (t = 0)
e	EULER’s number

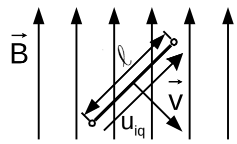
Series connection of capacitors	**Parallel connection of capacitors**
 $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$	 $C = C_1 + C_2 + \ldots + C_n$
$U = U_1 + U_2 + \ldots + U_n$	$U = U_1 = U_2 = \ldots = U_n$

<u>Magnetic field</u>

magnetic strength of field H	for the field outside of an direct conductor with the current I, apply to: $H = \frac{I}{2\pi r}$ $H = \frac{I}{4\pi \cdot r}(\cos\alpha_1 - \cos\alpha_2)$ for the field inside of an long coil with the current I, apply to: $H = \frac{N \cdot I}{l}$ for homogeneous magnetic field, apply to: $H = \frac{\Theta}{s}$	I current density r distance from conductor  N winding number of the coil l length of the coil
magnetic flux density B (magnetic induction)	$\vec{B} = \mu_0 \cdot \mu_r \cdot \vec{H}$	
permeability μ	$\mu = \mu_0 \cdot \mu_r$ for vacuum, apply to: $\mu_r = 1$	
magnetic flux Φ	$\Phi = \int \vec{B}\, d\vec{A}$ for $\vec{B}$ = constant and $\vec{B} \parallel \vec{A}$ apply to: $\Phi = B \cdot A$	Θ magnetomotive force s circle of an area μ_0 permittivity (vacuum, p.14) μ_r permeability A area
magnetic Voltage V	$V = \int_{P_1}^{P_2} \vec{H}(s)\, d\vec{s}$ under the condition of homogeneous field, apply to: $V = \vec{H} \cdot \vec{s}$	
magnetic resistor R_m	$R_m = \frac{l}{\mu_0 \cdot \mu_r \cdot A}$ $R_m = \frac{V}{\Phi}$	s distance l lenght A area Q charge v speed l length of the conductor
force of an moved electron F_L (LORENTZ's force)	$\vec{F}_L = Q \cdot \vec{v} \times \vec{B}$ under the condition $\vec{v} \perp \vec{B}$ apply to: $F_L = Q \cdot v \cdot B$	
force F to a conductor with current I	$\vec{F} = l \cdot \vec{I} \times \vec{B}$ under the condition $\vec{I} \perp \vec{B}$ apply to: $F = l \cdot I \cdot B$	
energy E of the magneticfield of a coil with current I	$E = \frac{1}{2} L \cdot I^2$	L inductivity of the coil

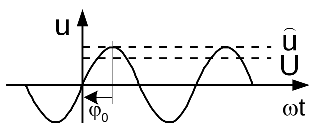
Electromagnetic field

<table>
<tr>
<td>Induction law</td>
<td>$u_{iq} = \frac{d\Phi}{dt}$
under the condition of steady change of the magnetic field and $\vec{B} \perp \vec{A}$
for a coil, apply to:
$u_{iq} = N\frac{\Delta(B \cdot A)}{\Delta t}$
for an moved conductor $\vec{v} \perp \vec{B}$ apply to:
$u_{iq} = v \cdot B \cdot \ell$</td>
<td rowspan="3">U_i induced voltage
Φ magnetic flux
N number of windings
t time
B magnetic flux density
A area


v speed of the conductor
l length of the conductor or the coil


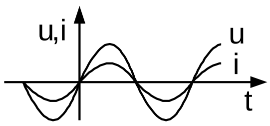
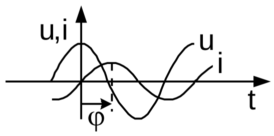
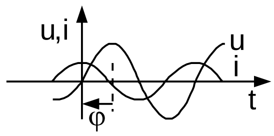
I current intensity
μ_0 permittivity (vacuum, p.14)
μ_r permeability
A area</td>
</tr>
<tr>
<td>self-induced voltage
in a coil</td>
<td>$u = L \cdot \frac{dI}{dt}$
under the condition of steady change of the current, apply to:
$u = L \cdot \frac{\Delta I}{\Delta t}$</td>
</tr>
<tr>
<td>inductivity L of an coil</td>
<td>for an long coil, apply to:
$L = \frac{\mu_0 \cdot \mu_r \cdot N^2 \cdot A}{l}$</td>
</tr>
</table>

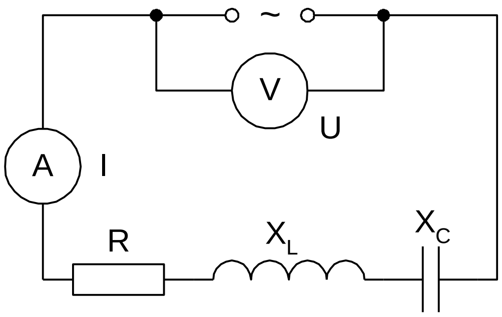
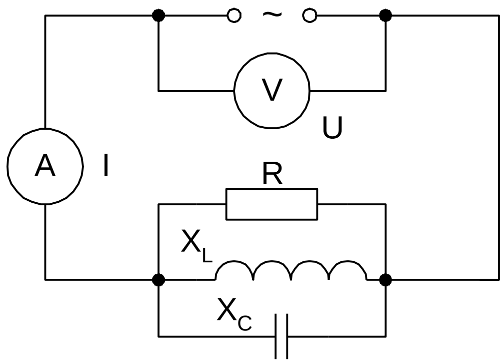
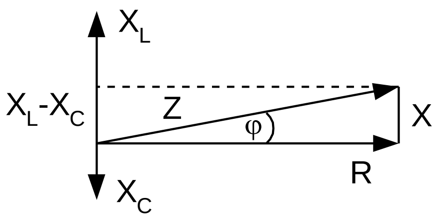
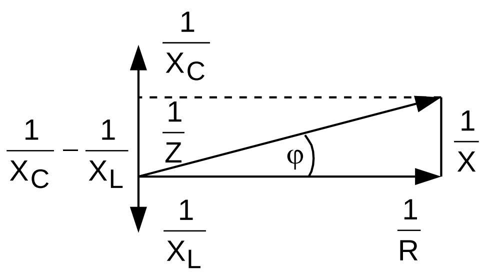
Alternating current circuit

<table>
<tr>
<td>current density i in alternating current circuit</td>
<td>momentary value: $i = \hat{I} \cdot \sin(\omega \cdot t + \varphi_0)$
root-mean-square value:
$I = \frac{1}{\sqrt{2}}\,\hat{i} \approx 0{,}7\,\hat{i}$</td>
<td rowspan="5">ω angular frequency
i momentary value
t time
$\hat{i}$ peak value
I root-mean-square value
φ_0 lag angle
u momentary value
$\hat{u}$ peak value
U root-mean-square value


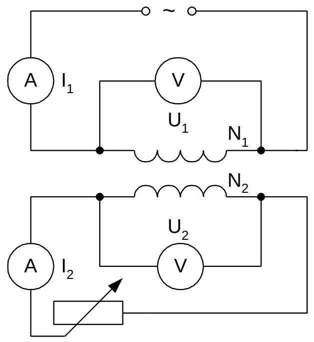
$\cos\varphi$ power factor
φ lag angle</td>
</tr>
<tr>
<td>voltage u in alternating current circuit</td>
<td>momentary value:
$u = \hat{u} \cdot \cos(\omega t + \varphi_0)$
root-mean-square value:
$U = \frac{1}{\sqrt{2}}\,\hat{u} \approx 0{,}7\,\hat{u}$</td>
</tr>
<tr>
<td>apparent power S</td>
<td>$S = U \cdot I$</td>
</tr>
<tr>
<td>effective power P</td>
<td>$P = U \cdot I \cdot \cos\varphi$</td>
</tr>
<tr>
<td>reactive power Q</td>
<td>$Q = U \cdot I \cdot \sin\varphi$</td>
</tr>
</table>

<u>Resistors in alternating current circuit</u>

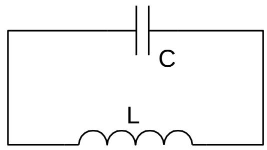
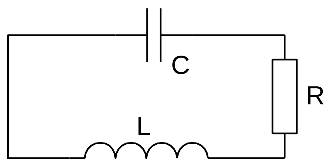
OHM's resistor R $R = \dfrac{U}{I}$	resistance of inductivity X_L $X_L = \dfrac{U}{I}$	resistance of capacitor X_C $X_C = \dfrac{U}{I}$
for a metallic conductor under the condition ϑ = constant, apply to: $R = \dfrac{\rho \cdot l}{A}$	for a coil, apply to: $X_L = \omega \cdot L$	for a capacitor, apply to: $X_C = \dfrac{1}{\omega \cdot C}$
		

	Series connection of R, X_L and X_C	**Parallel connection of R, X_L und X_C**
circuit		
phasor diagram		
reactive resistance X (reactance)	$X = \omega \cdot L - \dfrac{1}{\omega \cdot C}$	$\dfrac{1}{X} = \omega \cdot C - \dfrac{1}{\omega \cdot L}$
apparent resistance Z (impedance)	$Z = \sqrt{R^2 + X^2}$	$\dfrac{1}{Z} = \sqrt{\dfrac{1}{R^2} + \dfrac{1}{X^2}}$
lag tan φ	$\tan\varphi = \dfrac{X_L - X_C}{R}$	$\tan\varphi = R\left(\dfrac{1}{X_C} - \dfrac{1}{X_L}\right)$

Transformation

voltage transformation for an transformer without losses	under the condition $I_2 \to 0$ (no-load), apply to: $\frac{U_1}{U_2} = \frac{N_1}{N_2}$	
current transformation for an transformer without losses	under the condition $I_2 \to \infty$ (short), apply to: $\frac{I_1}{I_2} = \frac{N_2}{N_1}$	
transformation ratio ü	$ü = \frac{N_1}{N_2}$	U voltage I current intensity N number of windings P power P_v power loss φ lag angle between current and voltage P_{out} power output P_{in} power input
power transformation	$P_1 = P_2 + P_V$ $U_1 \cdot I_1 \cdot \cos\varphi_1 = U_2 \cdot I_2 \cdot \cos\varphi_2 + P_V$ under the conditions of heavy load, without losses and $\varphi_1 = \varphi_2$ apply to: $U_1 \cdot I_1 = U_2 \cdot I_2$	
effectiveness η of an transformer	$\eta = \frac{P_{out}}{P_{in}}$	

Electromagnetic oscillation

THOMSON's oszillation law	$T = 2\pi \cdot \sqrt{L \cdot C}$	T period L inductivity C capacity
frequency f of an electrical oscillating circuit (without attenuation)	under the condition of a free oscillation without attenuation (R=0), apply to: $f = \frac{1}{2\pi\sqrt{L \cdot C}}$	
natural frequency f of an electrical oscillating circuit (with attenuation)	under the condition of a free oscillation, apply to: $f = \frac{1}{2\pi}\sqrt{\frac{1}{L \cdot C} - \frac{R^2}{L^2}}$	R OHM's resistor L inductivity C capacity 
damping coefficient δ	$\delta = \frac{R}{2L}$	f natural frequency f_e exciter frequency
resonance condition	$f = f_e$	

Characteristics and units

characteristik	symbol	unit		Conversion between the units	
damping coefficient	δ	per second	s^{-1}	$1s^{-1}$	$= 60\ min^{-1}$
work	W, A	joule newtonmeter watt-second kilowatt-hour	J $N \cdot m$ $W \cdot s$ $kW \cdot h$	1 J $1\ kW \cdot h$	$= 1\ kg \cdot m^2 \cdot s^{-2}$ $= 1\ N \cdot m$ $= 1\ W \cdot s$ $= 3{,}6 \cdot 106\ W \cdot s$
illuminance	E	lux	lx	1 lx	$= 1\ lm \cdot m^{-2}$
acceleration	a, g	meter per square-second	$m \cdot s^{-2}$	$1\ m \cdot s^{-2}$	$= 1\ N \cdot kg^{-1}$
reactive power	Q	voltampere reactive	VAr	1 VAr	= 1 var
density (of aggregate)	ρ	kilogramme per cubic meter gramme per cubic centimeter	$kg \cdot m^{-3}$ $g \cdot cm^{-3}$	$1\ kg \cdot m^{-3}$ $1\ g \cdot cm^{-3}$	$= 10^{-3} g \cdot cm^{-3}$ $= 10^3\ kg \cdot m^{-3}$
torque	M	newtonmeter	$N \cdot m$	$1\ N \cdot m$	$= 1\ kg \cdot m^2 \cdot s^{-2}$
speed (rate of revolutions)	n	per second	s^{-1}	$1\ s^{-1}$	$= 60\ min^{-1}$
magnetomotive force	Θ	ampere	A		
breakdown strength	E_d	volt per meter	$V \cdot m^{-1}$		
energy	W	joule newtonmeter watt-second electron volt	J $N \cdot m$ $W \cdot s$ eV	1 J 1 eV	$= 1\ kg \cdot m^2 \cdot s^{-2}$ $= 1\ N \cdot m$ $= 1\ W \cdot s$ $= 1{,}602 \cdot 10^{-19}\ J$
gravitational acceleration	g	meter per square-second	$m \cdot s^{-1}$	$1\ m \cdot s^{-1}$	$= 1\ N \cdot kg^{-1}$
electrical field strength	E	volt per meter	$V \cdot m^{-1}$	$1\ V \cdot m^{-1}$	$= 1 kg \cdot m \cdot s^{-3} \cdot V^{-1}$
electrical flux	ψ	coulomb	C	1 C	$= 1\ A \cdot s$
magnetical flux	ϕ	weber	Wb	1 Wb	$= 1\ V \cdot s$
magnetical flux density (magnetic induction)	B	tesla	T	1 T	$= 1\ Wb \cdot m^{-2}$ $= 1\ V \cdot s \cdot m^{-2}$ $= 1\ N \cdot m^{-1} \cdot A^{-1}$

characteristic	**symbol**	**unit**		**Conversion between the units**	
frequency	f	hertz	Hz	1 Hz	= 1 s^{-1}
speed (propagation speed)	v c	meter per second kilometer per hour hitch	$m \cdot s^{-1}$ $km \cdot h^{-1}$ kn	1 $m \cdot s^{-1}$ 1 $km \cdot h^{-1}$ 1 kn	= 3,6 $km \cdot h^{-1}$ = 0,28 $m \cdot s^{-1}$ = 1 $sm \cdot h^{-1}$ = 1852 $m \cdot h^{-1}$
inductivity	L	henry	H	1 H	= 1 $Wb \cdot A^{-1}$ = 1 $m^2 \cdot kg \cdot s^{-2} \cdot A^{-2}$
capacity	C	farad	F	1 F	= 1 $A \cdot s \cdot V^{-1}$
force	F	newton kilopond	N kp	1 N 1 kp	= 1 $kg \cdot m \cdot s^{-2}$ = 1 $J \cdot m^{-1}$ = 9,81 N
angular frequency	ω	per second	s^{-1}	1 s^{-1}	= 60 min^{-1}
electrical charge	Q	coulomb	C	1 C	= 1 $A \cdot s$
power	P	watt	W	1 W	= 1 J · s-1 = 1 $V \cdot A$ = 1 $kg \cdot m^2 \cdot s^{-3}$ = 1 $N \cdot m \cdot s^{-1}$
power factor	$\cos\varphi$		1		
electrical conductivity	γ	siemens per meter	$S \cdot m^{-1}$	1 $S \cdot m^{-1}$	= 1 $\Omega^{-1} \cdot m^{-1}$ = $10^{-6} m \cdot \Omega$ -1·mm^{-2}
electrical conductance	G	siemens	S	1 S	= 1 Ω^{-1}
light density	L	candela per square-meter	$cd \cdot m^{-2}$		
period (of oszillation)	T	second	s	mentioned below (time)	
electical potential	φ	volt	V		
electrical voltage (potential difference)	U,u	volt	V	1 V	= 1 $kg \cdot m^2 \cdot s^{-3} \cdot V^{-1}$
magnetic voltage	V	ampere	A	1 A	= 1 $J \cdot Wb^{-1}$

characteristic	**symbol**	**unit**		**Conversion between the units**	
current	I, i	ampere	A	1 A	$= 1\ kg\cdot m^2\cdot s^{-3}\cdot V^{-1}$
heat (heat quantity)	Q	joule kalorie	J cal	1 J 1 cal	$= 1\ N \cdot m$ $= 1\ kg \cdot m^2 \cdot s^{-2}$ $=1\ W \cdot s$ = 4,19 J
heat capacity	C_{th}	joule per kelvin	$J \cdot K^{-1}$		
heat conduction resistor	R_λ	kelvin per watt	$K \cdot W^{-1}$		
heat current	Φ_{th}	watt	W	1 W	$= 1\ J \cdot s^{-1}$
OHM's resistor	R	ohm	Ω	1 Ω	$= 1\ V \cdot A^{-1}$ $= 1\ S^{-1}$
resistance of inductivity	X_L	ohm	Ω	1 Ω	$= 1\ V \cdot A^{-1}$
resistance of capacitor	X_C	ohm	Ω	1 Ω	$= 1\ V \cdot A^{-1}$
magnetic resistor	R_m	per henry	H^{-1}	$1\ H^{-1}$	$= 1\ A \cdot Wb^{-1}$
angle	$\alpha, \beta,$ γ, φ, σ ..	radiant degree of angle	rad °	1 rad 1°	$= \frac{180°}{\pi} \approx 57{,}296°$ $= \frac{\pi}{180} rad \approx 0{,}01745 rad$
angular acceleration	α	per squar-second	s^{-2}	$1\ s^{-2}$	$= 3600 \cdot min^{-2}$ $= 1\ rad \cdot s^{-2}$
angular speed	ω	per second	s^{-1}	$1\ s^{-1}$	$= 60\ min^{-1}$ $= 1\ rad \cdot s^{-1}$
efficiency	η		1 or %		
time (span, term)	t	second minute hour day year	s min h d a	 1 min 1 h 1 d 1 a	 = 60 s = 60 min = 3600 s = 24 h = 1440 min = 86400 s = 365 d oder 366 d

Physical constants

Constant	**symbol**	**value and unit**
permittivity (vacuum)	ε_0	$8{,}854 \cdot 10^{-12}$ As/Vm
permeability (vacuum)	μ_0	$1{,}257 \cdot 10^{-6}$ Vs/Am
elementary charge	e	$1{,}6021 \cdot 10^{-19}$ C
speed of light (vacuum)	c_0	$2{,}99792 \cdot 10^{8}$ m/s
electron mass (at rest)	m_e	$9{,}109 \cdot 10^{-31}$ kg
proton mass (at rest)	m_p	$1{,}6725 \cdot 10^{-24}$ g
neutron mass (at rest)	m_n	$1{,}6748 \cdot 10^{-24}$ g
Bolzmann constant	k	$1{,}381 \cdot 10^{-23}$ J/K
Planck constant	h	$6{,}626 \cdot 10^{-34}$ Js
gravitational constant	G	$6{,}673 \cdot 10^{-14}$ $M^3 / (g \cdot s^2)$
gravitational acceleration	g	9,80665 m/s^2
absolute zero of thermodynamical temperature	T_0	-273,15 °C
Loschmidt number	L	$6{,}023 \cdot 10^{23}$ molecules/mol